STAT 2593

Lecture 035 - The Two Sample t Test and Confidence Interval

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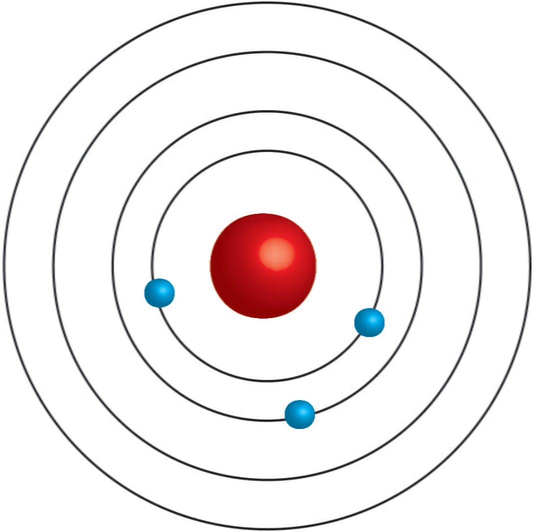
The Two Sample t Test and Confidence Interval

Learning Objectives

1. Construct hypothesis tests and confidence intervals for two sample populations with small sample sizes and unknown variances.

2. Understand the pooled variance estimator under the assumption of equal variances.



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  - What if we wish to conduct inference regarding two populations, with small sample sizes, and unknown variances.
  - We form the standard estimator, replacing the variances with the variance estimates,

$$Z = \frac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}}.$$

Distribution of the Test Statistic

- This will follow be follow an approximate t_ν distribution with

$$\nu = \frac{\left(\frac{s_1^2}{n} + \frac{s_2^2}{m}\right)}{\frac{(s_1^2/n)^2}{n-1} + \frac{(s_2^2/m)}{m-1}^2}.$$

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  - In practice, we use $\nu = \min\{n-1, m-1\}$.

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- Hypothesis tests of the form $H_0 : \mu_1 - \mu_2 = \Delta_0$ versus a (one- or two-sided) alternative can be tested in the usual way with the corresponding t distribution.
- This applies for any $m, n > 1$.
  - The procedure we previously saw, using normal approximations, applies only when σ_1^2 and σ_2^2 are known or if m and n are both large enough.

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- This will have a t distribution with $m + n - 2$ degrees of freedom.

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  - If this ratio falls between 0.5 and 2, a pooled estimator may be appropriate.
  - Outside of this range is generally not advisable to use this estimator.
- If there is *no good reason* to use a pooled estimator, don't.

Summary

- When testing a difference of means on two populations which have unknown variance and small sample size, we can use an approximate t distribution.

- If the variances between the two populations are known to be equal, then a pooled variance estimator can be used instead.